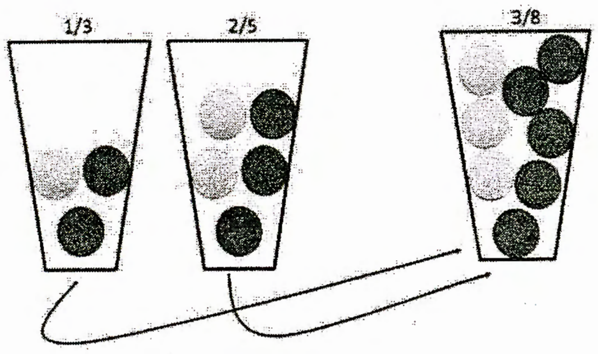
Fractions
Activity 6
Math-T101 Spring 2014

Name: Serife

Problem 1. *A student is wondering why $\frac{1}{3}+\frac{2}{5}$ is not equal to $\frac{3}{8}$. He presents the following figure as his reasoning. Explain to him where he is making a mistake and what he should do to fix it.*

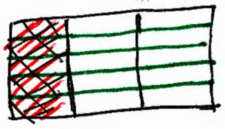


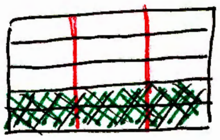
Why isn't it 3/8?

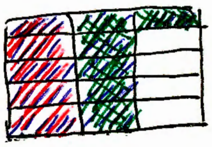
what the student did is;

$$\frac{1}{3}+\frac{2}{5}=\frac{1+2}{3+5}=\frac{3}{8}$$

Not true;

$\frac{1}{3}=$ 

$\frac{2}{5}=$ 

$\frac{1}{3}+\frac{2}{5}=$ 

$$\Downarrow$$

$$\frac{11}{15}$$

Note that my whole unit (3x5 rectangle) did not change and the parts that I added (5 parts + 6 parts) are in the same size.

Problem 2. *What is a* **whole unit***?*

A whole unit is 1

So; $1=\frac{3}{3}=\frac{5}{5}=\frac{8}{8}=\frac{a}{a}$

Problem 3. *What is a* **fractional unit***?*

A fractional unit OR unit fraction is $\frac{1}{a}$ because

$$\underbrace{\frac{1}{a}+\frac{1}{a}+\cdots+\frac{1}{a}}_{a\text{ times}}=\frac{a}{a}=1$$

(*) $\frac{a}{b}$: $a \rightarrow$ numerator, $b \rightarrow$ denominator

(*) $\frac{a}{b}$: $a \rightarrow$ how many fractional unit selected, $b \rightarrow$ how many parts your whole unit is divided into

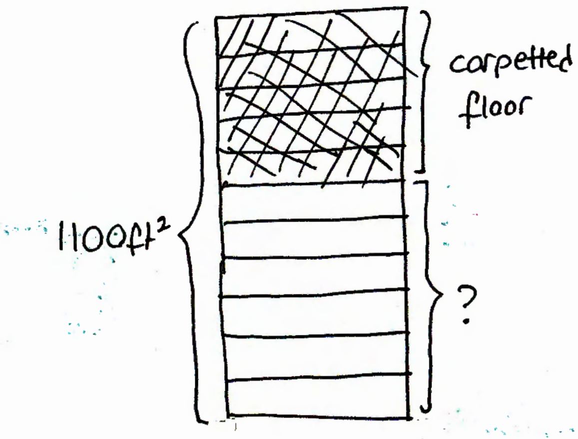
Then, any fraction can be written as a multiple of fractional unit

Defn. of fraction: $\frac{a}{b}=a\cdot\underbrace{\frac{1}{b}}_{\text{fractional unit}}$

We use the following models to illustrate fractions:

1. Area model

Problem 4. *(Teacher's Solution)* *The floor of Milli's house is 1100 square feet. She has $\frac{5}{11}$ of the floor carpeted, while the rest is hardwood flooring. How many square feet consist of hardwood flooring?*



11 units $= 1100\ ft^2$

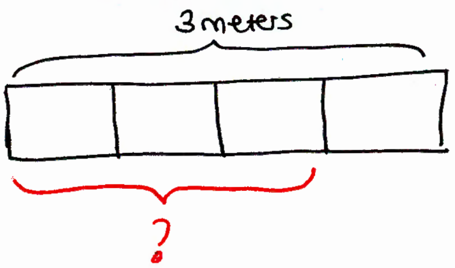
1 unit $= 100\ ft^2$

6 units $= 600\ ft^2$

$600 ft^2$ is hardwood flooring.

2. Linear measurement model (bar diagram or number line)

Problem 5. *(Teacher's Solution)* *Peter had a board 3 meters long. He used $\frac{3}{4}$ of its length as a bookshelf. How long was the bookshelf?*



4 units $= 3$ meters

1 unit $= \frac{3}{4}$ meters

3 units $= 3 \cdot \frac{3}{4}$ meters

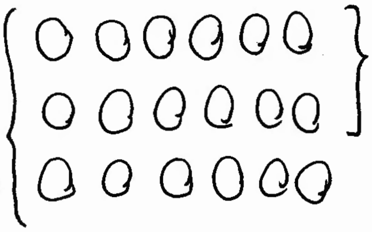
$= \frac{9}{4}$ meters

Bookshelf was $\frac{3}{4}$ meters long.

3. Set model

Problem 6. *(Teacher's Solution)* *Lidia has 18 apples. She gives $\frac{2}{3}$ of the apples to her neighbor. How many apples did she give to her neighbor?*

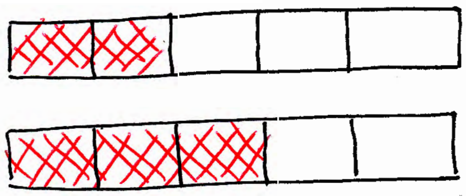
18 apples



$\frac{2}{3}$ of the apples = ? → $\frac{2}{3}$ of the 18 apples = 12 apples

Lidia gave 12 apples to her neighbor.

Problem 7. *Which is bigger: $\frac{2}{5}$ or $\frac{3}{5}$?*



$= \frac{2}{5}$

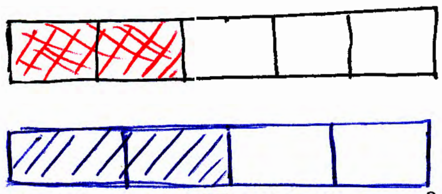
$= \frac{3}{5}$

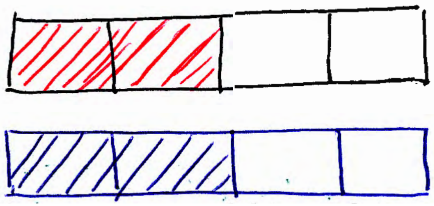
$\frac{2}{5} < \frac{3}{5}$

Problem 8. *Which is bigger: $\frac{2}{5}$ or $\frac{2}{4}$?*

$\frac{2}{5} =$

$\frac{1}{2} =$





$= \frac{2}{4}$

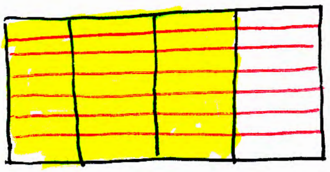
$= \frac{1}{2}$

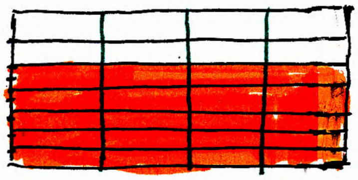
$\frac{2}{5} < \frac{1}{2}$

$\frac{2}{4} = \frac{1}{2}$

$\frac{2}{5} < \frac{2}{4}$

Problem 9. *Which is bigger: $\frac{3}{4}$ or $\frac{5}{7}$?*

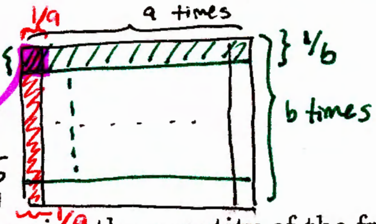




$\frac{3}{4} > \frac{5}{7}$

(21 small units) (20 small units)

Lemma $\Rightarrow \frac{1}{a}\cdot\frac{1}{b} = \frac{1}{a\cdot b}$ $\longrightarrow$ pictorial proof



Area of this small rectangle $= \frac{1}{a}\cdot\frac{1}{b}$
It is also $\frac{1}{a\cdot b}$ from big rect.

Defn of fraction $\Rightarrow a\cdot\frac{1}{b} = \frac{a}{b}$

multiplicative inverse prop.
$\Downarrow$
$a\cdot\frac{1}{a} = 1$

We can change the fractional unit of a fraction, while keeping the quantity of the fraction the same. This is called "renaming the fraction" or "equivalent fractions".

Rule 1. $\frac{a}{b} = \frac{an}{bn}$ *if a,b,n are whole numbers, and b and n are nonzero.*

$$\frac{an}{bn} = an\cdot\frac{1}{bn} = \left(a\cdot\frac{1}{b}\right)\left(n\cdot\frac{1}{n}\right) = a\cdot\frac{1}{b}\cdot 1 = a\cdot\frac{1}{b} = \frac{a}{b}$$

(DF) (any-order prop.) (Multip. Inverse prop.) (multip. identity) (DF)

To add and subtract fractions with the same fractional unit, what do we do?
For example, $\frac{1}{5}+\frac{2}{5}=$ $\frac{1+2}{5} = \frac{3}{5}$

Rule 2. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ *and* $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$ *if a,b,c are whole numbers, and* $b \neq 0$.

$$\frac{a}{b} \mp \frac{c}{b} = a\cdot\frac{1}{b} \mp c\cdot\frac{1}{b} = (a\mp c)\cdot\frac{1}{b} = \frac{a\mp c}{b}$$

(D.F) (Distributive prop.) (D.F)

When the fractional units are not the same (that is, the denominators are different), what can we do?
For example, $\frac{2}{3}$-$\frac{1}{5}$=

(R.1) $\frac{2}{3} = \frac{2\cdot 5}{3\cdot 5} = \frac{10}{15}$
$\frac{1}{5} = \frac{1\cdot 3}{5\cdot 3} = \frac{3}{15}$

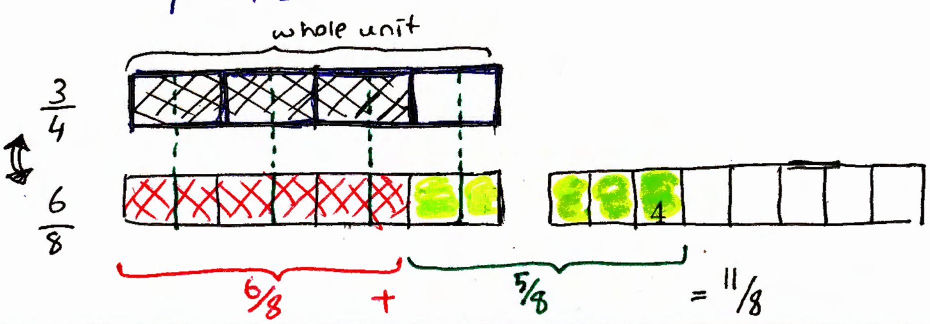
$\frac{2}{3} - \frac{1}{5} = \frac{10}{15} - \frac{3}{15} = \frac{10-3}{15} = \frac{7}{15}$
(R.2)

Problem 10. *Find* $\frac{3}{4} + \frac{5}{8}$ *without using SCA.*

$$\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{6+5}{8} = \frac{11}{8}$$
(R.2)

$\downarrow$

$$\frac{3}{4} = \frac{3\cdot 2}{4\cdot 2} = \frac{6}{8}$$ (R.1)



Problem 11. *Find $\frac{5}{6} - \frac{3}{4}$ without using SCA.*

$$\frac{5}{6} - \frac{3}{4}$$

$$\frac{5 \cdot 2}{6 \cdot 2} - \frac{3 \cdot 3}{4 \cdot 3} = \frac{10}{12} - \frac{9}{12} = \frac{10-9}{12} = \frac{1}{12}$$

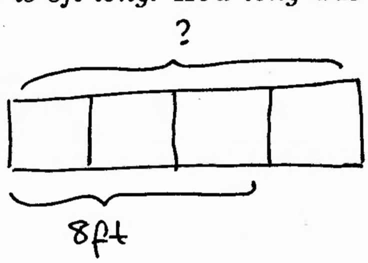
(R.1) (R.2)

Problem 12. *Find $\frac{1}{4} + \frac{5}{7}$ without using SCA.*

$$\frac{1}{4} + \frac{5}{7} = \frac{1 \cdot 7}{4 \cdot 7} + \frac{5 \cdot 4}{7 \cdot 4} = \frac{7}{28} + \frac{20}{28} = \frac{7+20}{28} = \frac{27}{28}$$

(R.1) (R.2)

Problem 13. *(Teacher's Solution)* *Vladimir used $\frac{3}{4}$ of the length of a board for a bookshelf. The bookshelf is 8ft long. How long was the board?*



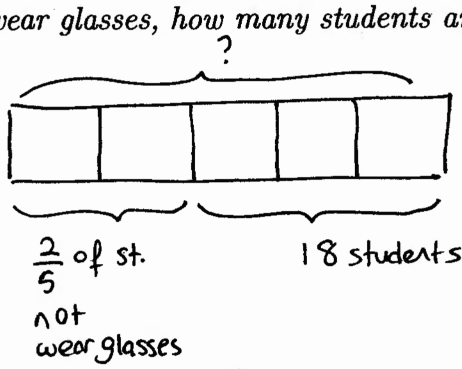
3 unit $= 8$ ft

1 unit $= \frac{8}{3}$ ft

4 units $= 4 \cdot \frac{8}{3} = \frac{32}{3}$ ft

The board was $\frac{32}{3}$ ft long.

Problem 14. *(Teacher's Solution)* *In a class $\frac{2}{5}$ of the students wear glasses. If 18 students do not wear glasses, how many students are there altogether?*



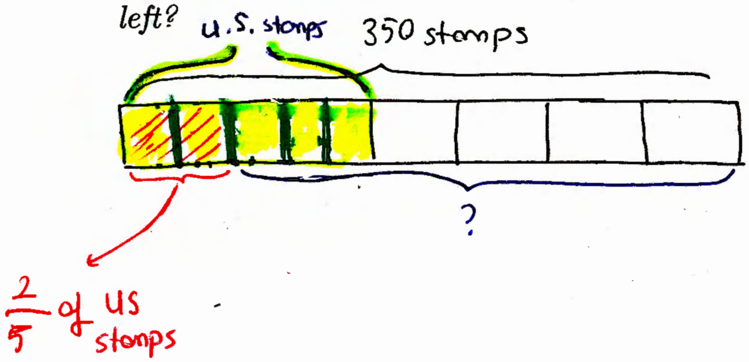
3 units $=$ 18 students

1 unit $=$ 6 students

5 units $=$ 30 students

There were 30 students altogether.

Problem 15. *(Teacher's Solution)* *Ali had 350 stamps. $\frac{3}{7}$ of his stamps were US stamps, while the rest were not. He gave $\frac{2}{5}$ of his US stamps to Ryan. How many stamps did he have left?*



350 stamps = 7 units

50 stamps = 1 unit

150 stamps = 3 units (u.s. stamps)

5 small (green) units = 150

1 small unit = 30

2 small units = 60 → 350 − 60 = 290

Ali had 290 stamps left

A mixed number is a whole number plus a fraction. For example, $2\frac{1}{5}$ is the same as $2+\frac{1}{5}$. An improper fraction is a fraction whose numerator is larger than its denominator, such as $\frac{10}{3}$.

Problem 16. $\frac{5}{11}+(\frac{4}{7}+2\frac{6}{11})$

$$\frac{5}{11}+\left(\frac{4}{7}+2\frac{6}{11}\right)=\left(\frac{5}{11}+2\frac{6}{11}\right)+\frac{4}{7}=2\frac{11}{11}+\frac{4}{7}$$

(Any-order property)

$$=3+\frac{4}{7}$$

$$=3\frac{4}{7}$$

Problem 17. *Calculate $3\frac{1}{8}-1\frac{5}{16}$ using mental math strategies. Then use that to find $1\frac{5}{16}-3\frac{1}{8}$.*

$$3\frac{1}{8}-1\frac{5}{16}=3\frac{2}{16}-1\frac{5}{16}$$

$$=2\frac{18}{16}-1\frac{5}{16}$$

$$=(2-1)+\left(\frac{18}{16}-\frac{5}{16}\right)$$

$$=1+\frac{18-5}{16}$$

$$=1+\frac{13}{16}=1\frac{13}{16}$$

$$1\frac{5}{16}-3\frac{1}{8}=-\left(3\frac{1}{8}-1\frac{5}{16}\right)$$

$$=-\left(1\frac{13}{16}\right)$$

$$=-1\frac{13}{16}$$

Fractions provide an answer (a quotient) to division problems involving whole number divided by another whole number. For example, $13 \div 2$ can be thought of as $\frac{13}{2}$ or $6\frac{1}{2}$.

Rule 3. $a \div b = \frac{a}{b}$ if a and b are whole numbers and $b \neq 0$.

Problem 18. *When asked how can 5 people divide 8 cupcakes, Nancy replied that they each get 1R3 cupcakes. Explain why her answer does not make sense in real life and how we should do this division.*

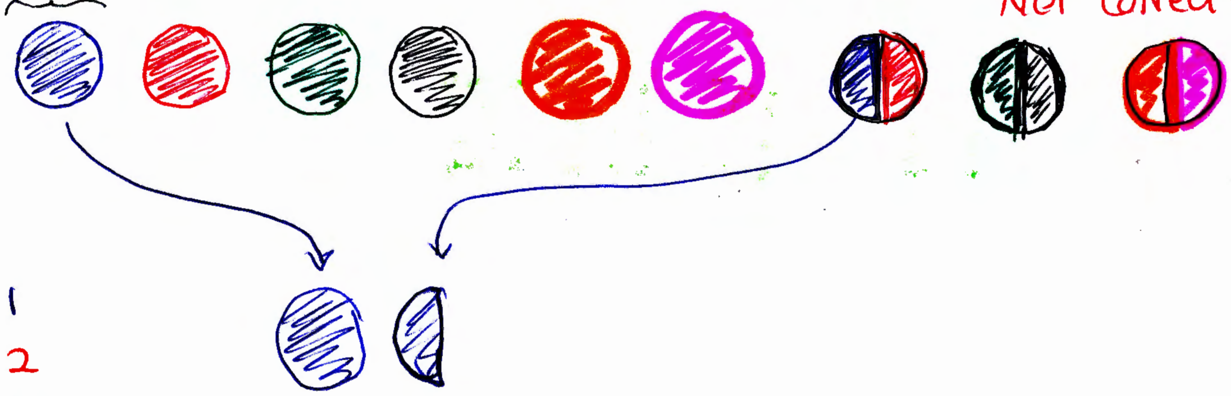
Nancy divided 9 cupcakes to 6 people, finding how much cake each person will get is a division problem;

$$9 \div 6 = \text{how much each will get} = \frac{9}{6} \neq 1R3$$

this doesn't make sense in real life because if this is the case than, each person will get 1 cupcake and 3 leftover cupcakes

$$\underbrace{\left[\left(\underset{\text{cake}}{1} + 3\text{ leftover}\right) \cdot 6 = \underset{\text{cakes}}{6} + \underset{\text{leftover cakes}}{18}\right] = 24 \text{ cakes}}_{\text{Not correct.}}$$

whole unit



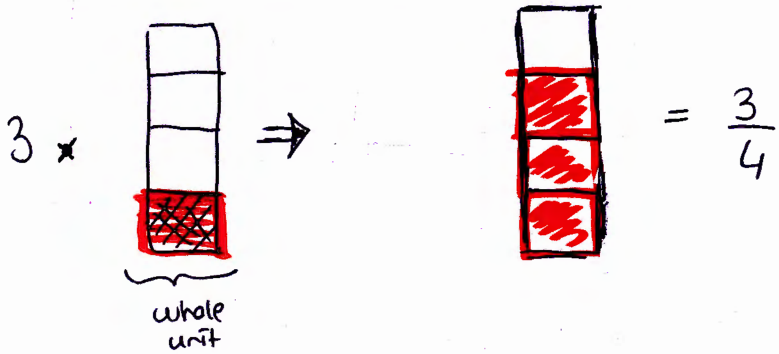
- person 1
- person 2
- person 3
- person 4
- person 5
- person 6

$$1 + \frac{1}{2} = 1\frac{1}{2} = \frac{9}{6}$$ each person will get

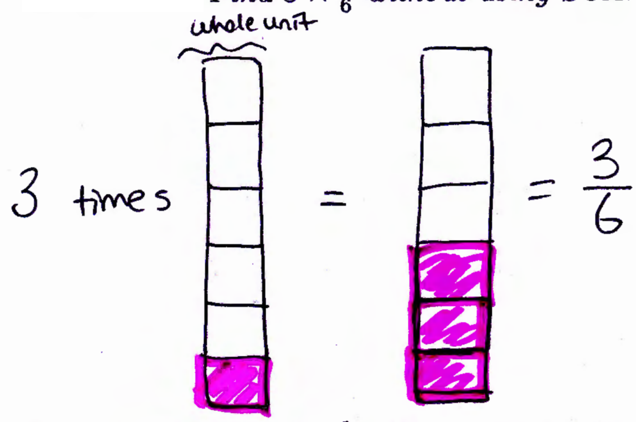
Multiplication of fractions

Whole number times a fraction

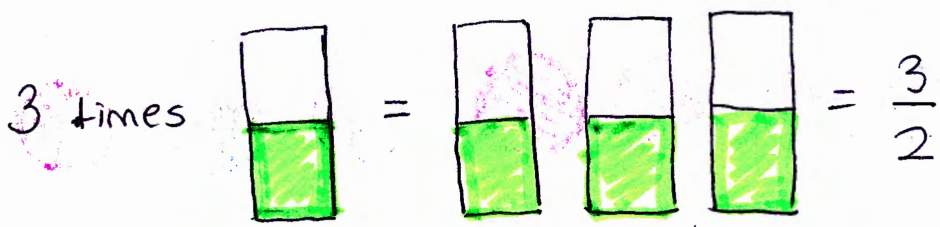
Problem 19. *Find* $3 \times \frac{1}{4}$ *without using SCA.*



Problem 20. *Find* $3 \times \frac{1}{6}$ *without using SCA.*



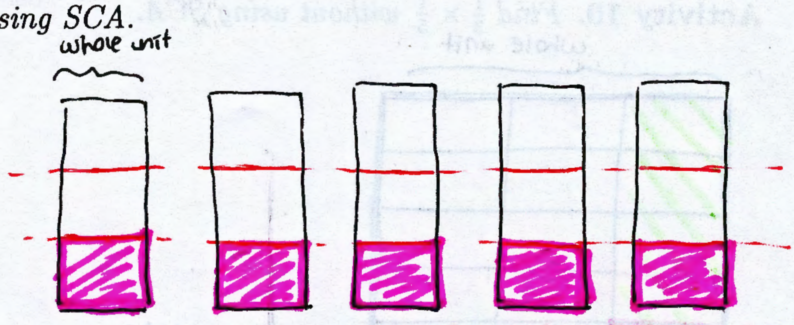
Problem 21. *Find* $3 \times \frac{1}{2}$ *without using SCA.*



Fraction times a whole number

Problem 22. *Find $\frac{1}{3} \times 5$ without using SCA.*

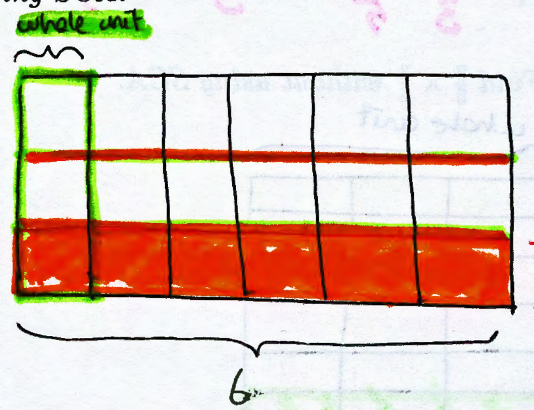
$\frac{1}{3} \times 5 = \frac{1}{3}$ of $5 \Rightarrow$



$\Rightarrow$ 5 times $\frac{1}{3} = \frac{5}{3}$

Problem 23. *Find $\frac{1}{3} \times 6$ without using SCA.*

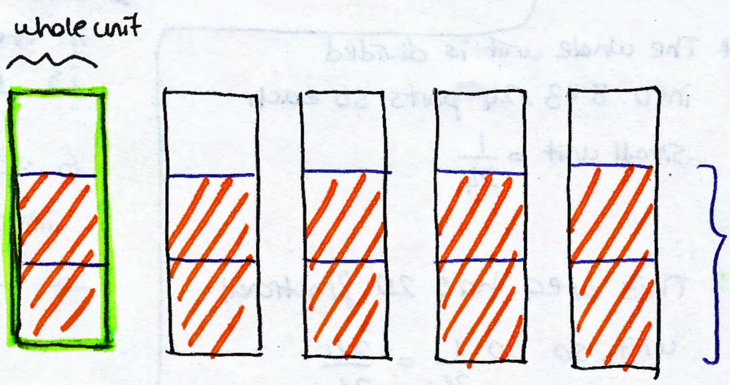
$\frac{1}{3} \times 6 = \frac{1}{3}$ of 6



$\frac{1}{3}$ of $6 = \frac{6}{3}$

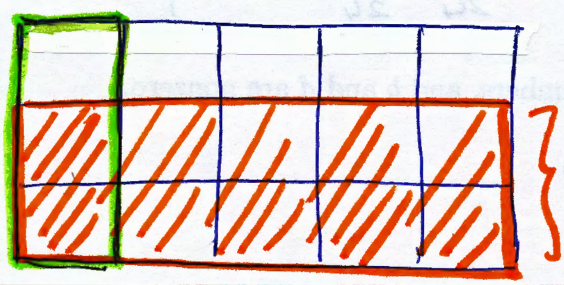
Problem 24. *Find $\frac{2}{3} \times 5$ without using SCA.*

$\frac{2}{3} \times 5 = \frac{2}{3}$ of 5



$\frac{10}{3}$

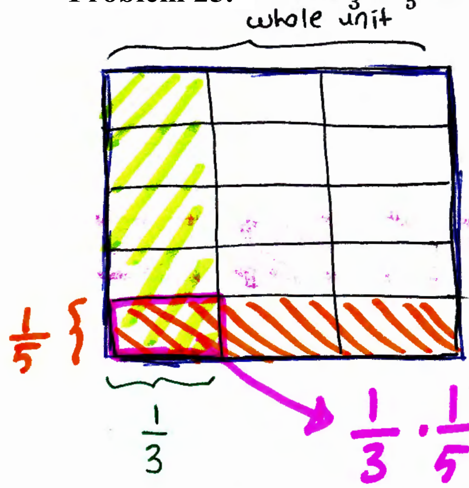
OR



$\frac{2}{3}$ of $5 = \frac{10}{3}$

Fraction times a fraction

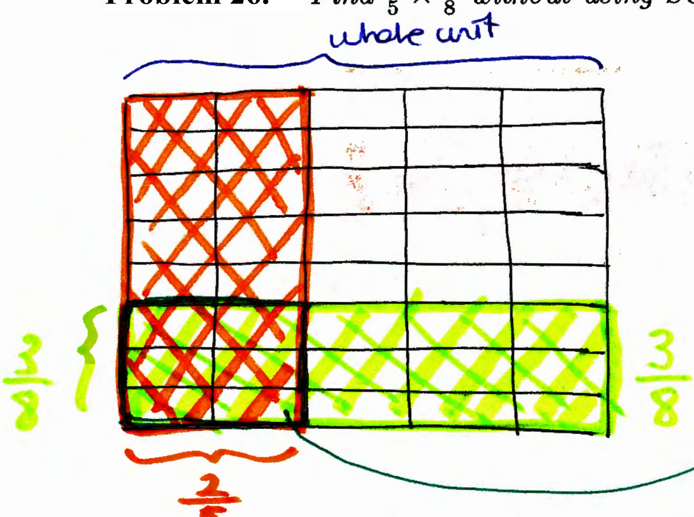
Problem 25. *Find $\frac{1}{3} \times \frac{1}{5}$ without using SCA.*



[Area of the rectangle which has the sides $\frac{1}{3}$ and $\frac{1}{5}$]

$\frac{1}{3} \cdot \frac{1}{5} = \frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ → note that whole unit is divided into 15 parts

Problem 26. *Find $\frac{2}{5} \times \frac{3}{8}$ without using SCA.*



$\underbrace{\frac{2}{5} \cdot \frac{3}{8}}_{\text{area of the rectangle which is } \frac{2}{5} \text{ by } \frac{3}{8}} = \frac{6}{40}$

6 → 6 fractional units in the intersection

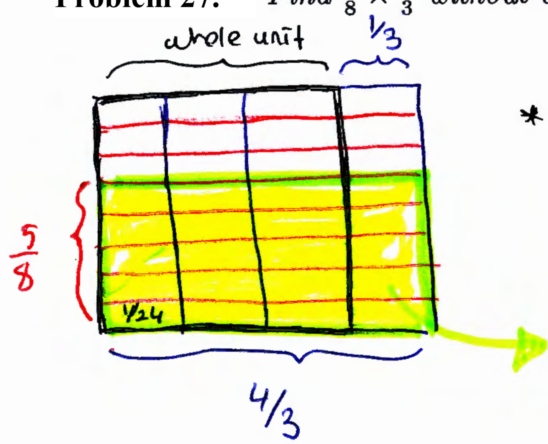
40 → whole unit is divided into 40 parts

each small parts in the whole unit is $\frac{1}{40}$ and 6 of those are in the intersection

so $\frac{2}{5} \cdot \frac{3}{8} = 6 \cdot \frac{1}{40}$

$= \frac{6}{40}$

Problem 27. *Find $\frac{5}{8} \times \frac{4}{3}$ without using SCA.*

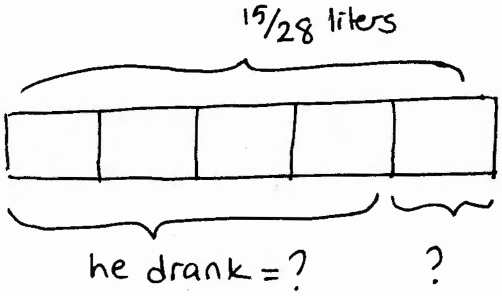


* The whole unit is divided into $8 \cdot 3 = 24$ parts so each small unit $= \frac{1}{24}$

This area has 20 fractional unit, so $20 \cdot \frac{1}{24} = \frac{20}{24}$

Rule 4. $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ if a,b,c,d are whole numbers, and b and d are nonzero.

Problem 28. *(Teacher's Solution)* *Steve had $\frac{15}{28}$ liters of coke. He drank $\frac{4}{5}$ of it. How much did he drink? How much coke is left?*



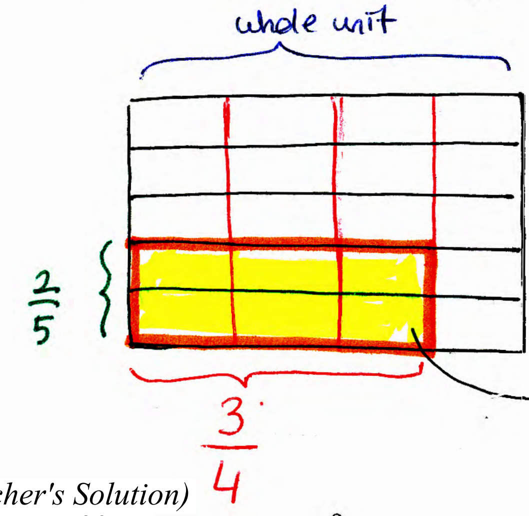
5 units = $\frac{15}{28}$ liters

1 unit = $\frac{3}{28}$ liters

4 units = $\frac{12}{28}$ liters

He drank $\frac{12}{28}$ liters and $\frac{3}{28}$ liters of coke is left.

Problem 29. *A rectangle measures $\frac{3}{4}$ yd by $\frac{2}{5}$ yd. Find its area.*

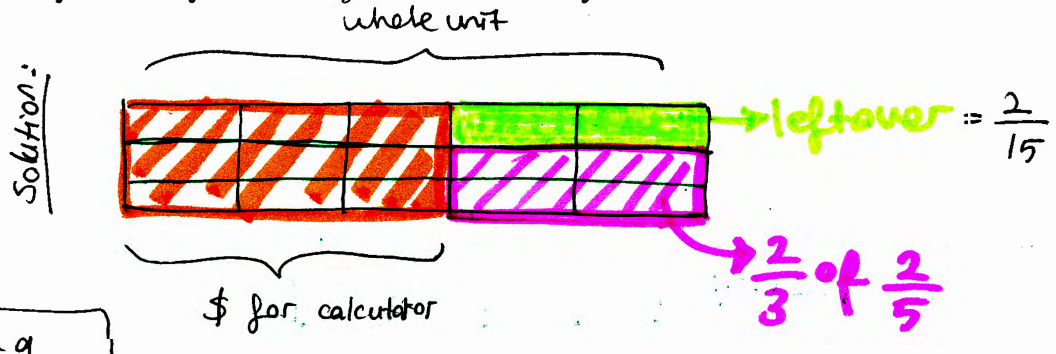


This is the rectangle which is $\frac{3}{4}$ by $\frac{2}{5}$

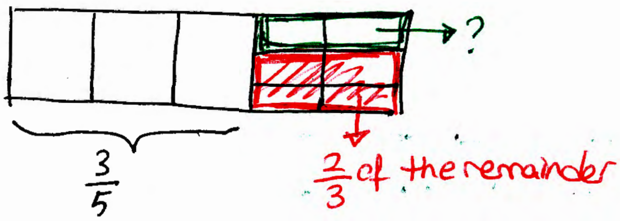
The area is $\frac{6}{20}$ yd^2

(Teacher's Solution)

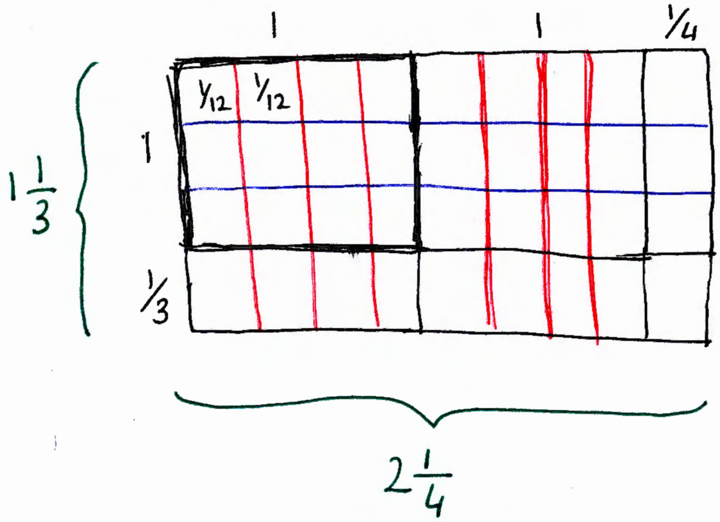
Problem 30. *Susan spent $\frac{3}{5}$ of her money on a calculator and $\frac{2}{3}$ of the remainder on a pen. What fraction of her money does she have left?*



Representation of the Question!



Problem 31. *Use the area model to find $2\frac{1}{4} \times 1\frac{1}{3}$. One common student error is to write $2\frac{1}{4} \times 1\frac{1}{3} = 2\frac{1}{12}$. Use the area model, and give a brief explanation which simultaneously shows both the error this student is making and what the correct solution is.*



* since 1 whole unit is divided into $3 \times 4 = 12$ small units. Each unit (fractional unit) is $\frac{1}{12}$

In the big rectangle $(2\frac{1}{4}$ by $1\frac{1}{3})$, there are 36 fractional units $\Rightarrow 36 \cdot \frac{1}{12} = \frac{36}{12} = 3$

1 Division of fractions-further discussion

What is a *partitive division*?

\# of groups is known
size of the group is unknown

interpretive question for $12 \div \frac{3}{4}$
$\frac{3}{4}$ groups of what size is 12?
OR 12 is $\frac{3}{4}$ groups of what size?

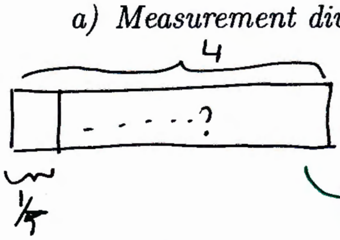
What is a *measurement division*?

size of the each group is known
\# of groups is unknown

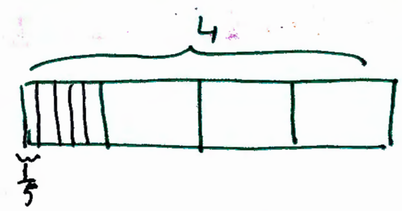
interpretive question for $\frac{5}{2} \div \frac{3}{4}$
How many $\frac{3}{4}$'s make $\frac{5}{2}$?
OR $\frac{5}{2}$ is how many $\frac{3}{4}$'s?

Problem 32. . *Illustrate the following with a bar diagram and solve the problem.*

a) Measurement division for $4 \div \frac{1}{5}$. → interpretive question: How many $\frac{1}{5}$'s make 4?
OR 4 is how many $\frac{1}{5}$'s?

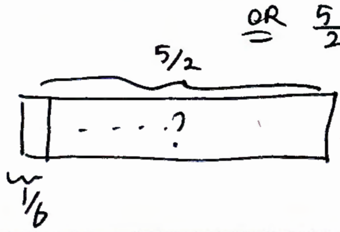


solution:

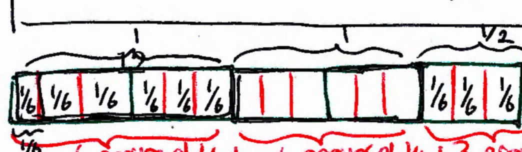


5 groups of 1/5 + 5 groups of 1/5 + 5 groups of 1/5 + 5 groups of 1/5 = 20 groups of $\frac{1}{5}$'s make 4

b) Measurement division for $\frac{5}{2} \div \frac{1}{6}$.
Interpretive question: How many $\frac{1}{6}$'s make $\frac{5}{2}$?
OR $\frac{5}{2}$ is how many 1/6's?



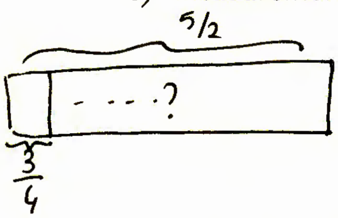
solution:



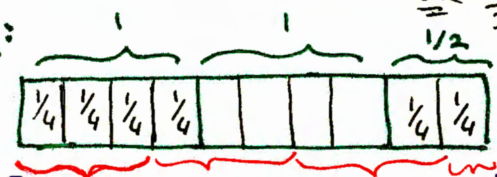
6 groups of 1/6 + 6 groups of 1/6 + 3 groups of 1/6 = 15 groups of 1/6

c) *Measurement division for* $\frac{5}{2} \div \frac{3}{4}$. Interpretive question: How many $\frac{3}{4}$'s make $\frac{5}{2}$?

OR $\frac{5}{2}$ is how many $\frac{3}{4}$'s?



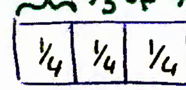
→ solution:



1 group of $\frac{3}{4}$ + 1 group of $\frac{3}{4}$ + 1 group of $\frac{3}{4}$ + $\frac{1}{3}$ of $\frac{3}{4}$ $= 3 + \frac{1}{3} = 3\frac{1}{3}$ groups of $\frac{3}{4}$ makes $\frac{5}{2}$.

* in detail

$\frac{1}{3}$ of $\frac{3}{4}$

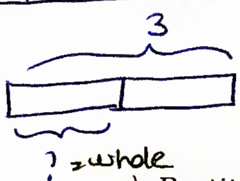


1 group of $\frac{3}{4}$

because 3 times $\frac{1}{4}$ makes $\frac{3}{4}$, so, $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$.

d) *Partitive division for* $3 \div 2$.

Interpretive Question: 2 groups of what size is 3?

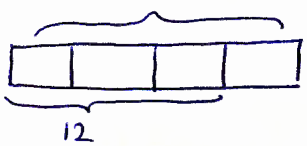


2 units = 3

1 unit = $\frac{3}{2}$

e) *Partitive division for* $12 \div \frac{3}{4}$. Interpretive Question

$\frac{3}{4}$ of what size is 12?

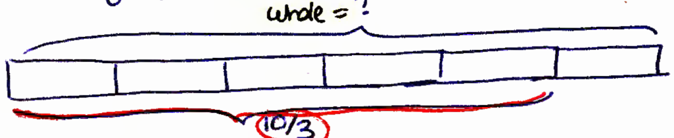


3 units = 12

1 unit = 4 → 4 units = 16

f) *Partitive division for* $\frac{10}{3} \div \frac{5}{6}$.

Interpretive Question: $\frac{5}{6}$ of what size is $\frac{10}{3}$?



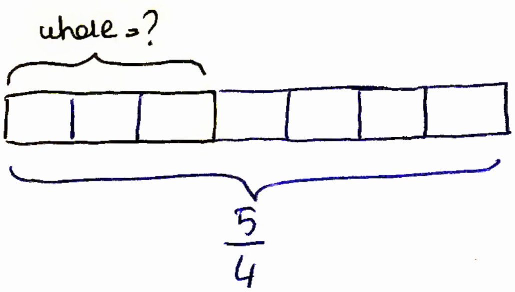
5 units = $\frac{10}{3}$) ÷5

1 unit = $\frac{2}{3}$) ×6

6 units = $\frac{12}{3}$

g) *Partitive division for* $\frac{5}{4} \div \frac{7}{3}$.

Interpretive Question: $\frac{7}{3}$ of what size is $\frac{5}{4}$?



÷7 (7 units = $\frac{5}{4}$

1 unit = $\boxed{\frac{5}{4} \div 7}$

1 unit = $\frac{5}{28}$

OR

7 units = $\frac{5}{4}$

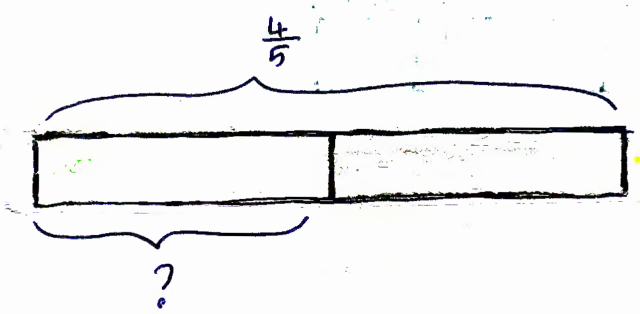
$\frac{1}{7} \cdot$ 7 units = $\boxed{\frac{5}{4} \cdot \frac{1}{7}}$

1 unit = $\frac{5}{28}$

3 units = $3 \cdot \frac{5}{28} = \frac{15}{28}$ $\frac{}{7}$

2 Partitive

Problem 33. *(Teacher's Solution)* *Noah picked $\frac{4}{5}$ pound of berries. He divided them evenly among 2 containers. How many pounds of berries did Noah put in each container?* → partitive division

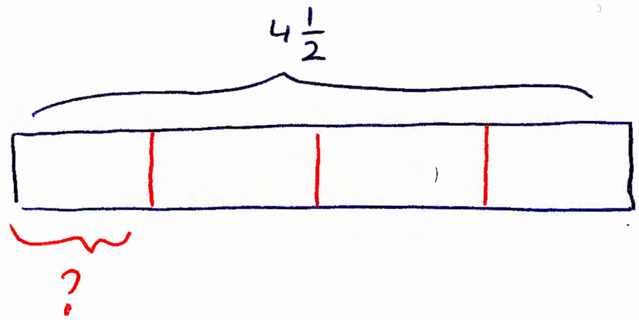


$2 \text{ units} = \frac{4}{5}$

$1 \text{ unit} = \frac{2}{5}$

Noah put $\frac{2}{5}$ pound of berries in each container.

Problem 34. *(Teacher's Solution)* *At the end of the day, there were $4\frac{1}{2}$ pizzas left over at a pizza place. The manager decided to divide it equally among the four workers. How much pizza does each get?*



$4 \text{ units} = 4\frac{1}{2}$

1st way

$4 \text{ units} = \frac{9}{2}$

$\frac{1}{4} \cdot 4 \text{ units} = \frac{9}{2} \cdot \frac{1}{4}$

$1 \text{ unit} = \frac{9}{8}$

2nd way

$4 \text{ units} = 4 + \frac{1}{2}$

$\frac{1}{4} \cdot 4 \text{ units} = \frac{1}{4} \cdot \left(4 + \frac{1}{2}\right)$

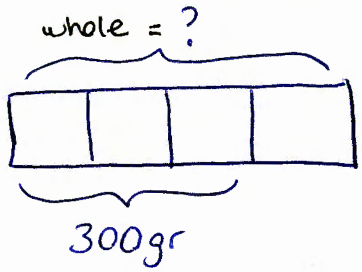
$1 \text{ unit} = \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot \frac{1}{2}$

$1 \text{ unit} = 1 + \frac{1}{8}$

$1 \text{ unit} = 1\frac{1}{8} = \frac{9}{8}$

Each worker will get $\frac{9}{8}$ pizza.

Problem 35. (*Teacher's Solution*) *Jenny cooked a pie and $\frac{3}{4}$ of it was left. She knows that this left-over pie weights 300 gr. How much was the weight of the whole pie?*



$$\begin{aligned} 3 \text{ units} &= 300 \\ 1 \text{ unit} &= 100 \\ 4 \text{ units} &= 400 \end{aligned}$$

$\div 3$

$\times 4$

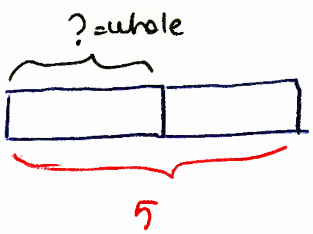
So, whole pie is 400gr.

✱ Division question for this problem is

$$300 \div \frac{3}{4} = ?$$

Interpretive question: $\frac{3}{4}$ of what size is 300?

Problem 36. *5 is 2 of what?*

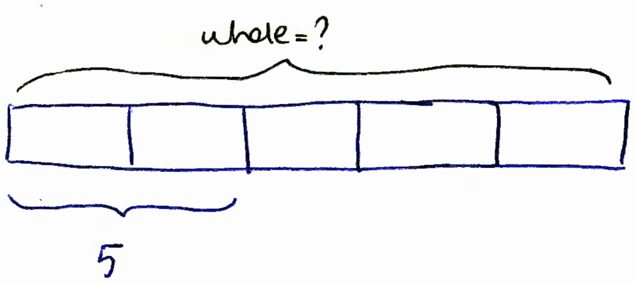


$5 \div 2 \Rightarrow$

$2 \text{ units} = 5$

$1 \text{ unit} = \frac{5}{2}$

Problem 37. *5 is $\frac{2}{5}$ of what?*



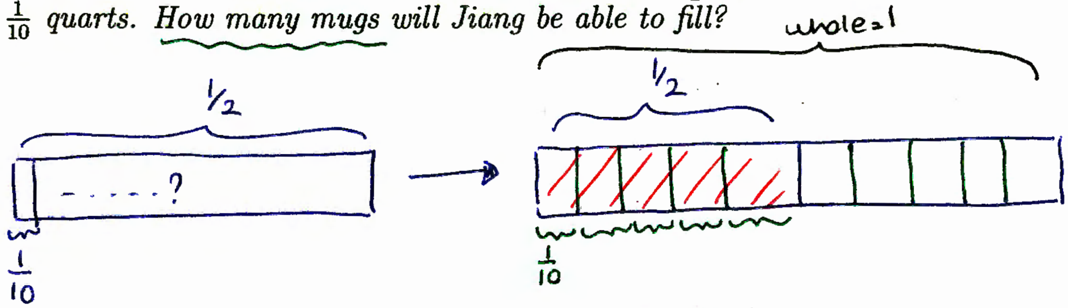
$$\begin{aligned} 2 \text{ units} &= 5 \\ 1 \text{ unit} &= \frac{5}{2} \\ 5 \text{ unit} &= \frac{25}{2} \end{aligned}$$

$\div 2$

$\times 5$

3 Measurement

Problem 38. (*Teacher's Solution*) *Jiang made $\frac{1}{2}$ quarts of hot chocolate. Each mug holds $\frac{1}{10}$ quarts. How many mugs will Jiang be able to fill?*



$1+1+1+1+1 = 5$ groups of $\frac{1}{10}$ makes $\frac{1}{2}$

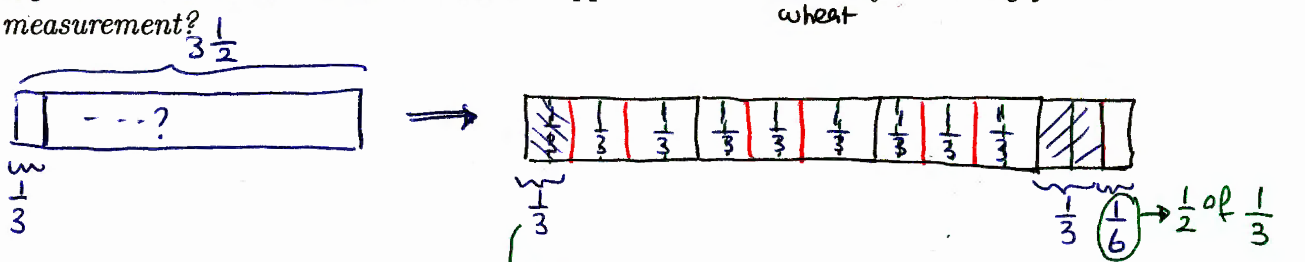
So, 5 mugs Jiang will be able to fill.

Problem 39. *The ingredients for a simple whole wheat bread are:*[1]

- *3 cups warm water (110 degrees F/45 degrees C)*
- *2 (.25 ounce) packages active dry yeast*
- *2/3 cup honey*
- *5 cups bread flour*
- *3 tablespoons butter, melted*
- *1 tablespoon salt*
- *3 1/2 cups whole wheat flour*
- *2 tablespoons butter, melted*

Jenny wants to make this bread, but all she has for measurement is a 1/3-cup measurement and a tablespoon (which is 1/16 of a cup). How can she measure the ingredients using her existing measurement tools? How could she approximate the ~~bread~~ wheat flour using just the 1/3-cup measurement?

wheat flour:

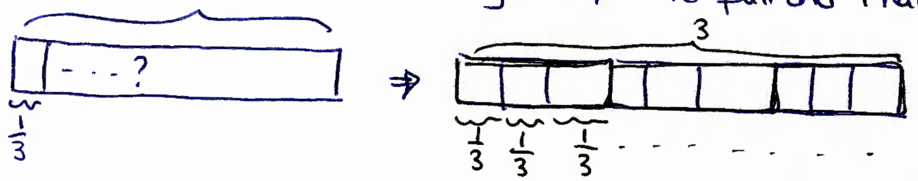


$1+1+1+1+1+1+1+1+1+1+\frac{1}{2} = 10\frac{1}{2}$ group of $\frac{1}{3}$

group of $\frac{1}{3}$ | group of $\frac{1}{3}$

Jenny will put 10 full and 1 half $\frac{1}{3}$-cups of flour

water: $3 \div \frac{1}{3} = ?$



⇒ 9 groups of $\frac{1}{3}$ makes 3

Jenny will put 9 $\frac{1}{3}$-cups of water.

yeast = ?

honey = ?

bread flour = ?

[1] http://allrecipes.com/Recipe/Simple-Whole-Wheat-Bread/Detail.aspx

Problem 40. *Make a word problem using the sharing concept of division involving fractions. Can you make a very different one? What modifications have you made to make a new problem?*

Sharing concept } Jack had $\frac{3}{5}$ lb of sugar, he wants to put it into 3 equal packages. What is the weight of each package in lb?

Problem 41. *Make a word problem using the measuring concept of division involving fractions. Can you make a very different one? What modifications have you made to make a new problem?*

Measurement concept } Alex made 12 cookies. He put $\frac{3}{4}$ of an egg on each cookie. How many eggs did he used?

Problem 42. *A student solved the problem $\frac{3}{4} \div \frac{1}{2}$ by "invert and multiply" as follows. How would you analyze student's work and give feedback to the student? Address your answer directly to the student.*

$$\frac{3}{4} \div \frac{1}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{4 \times 1}{3 \times 2} = \frac{4}{6} = \frac{2}{3}$$

Invert & multiply rule is not descriptive. That is, it does not explain the rule. It also does not explain the underlying meaning of fraction division. In this example, student inverted and multiplied the first fraction; however, $\frac{3}{4} \div \frac{1}{2}$ asks either how many $\frac{1}{2}$'s makes $\frac{3}{4}$ (measurement interpretation) or $\frac{1}{2}$ of what size/number is $\frac{3}{4}$ (partitive interpretation). Considering the second one, if $\frac{3}{4}$ is $\frac{1}{2}$ of a number (i.e. half of a number) then this number will be 2 times $\frac{3}{4}$ (i.e. $\frac{3}{4} \cdot 2 = \frac{3}{4} \cdot \left(\frac{2}{1}\right)$). "see invert & multiply rule"

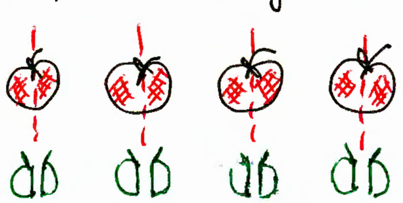
Problem 43. *Discuss the following statement with your group and decide if it is "true" or "false" and give convincing reasoning:* [2]

Division makes smaller (that is, in a division problem, the quotient must be less than the dividend). Note: the terms "quotient" and "dividend" are defined on p. 32 of the text.

on page 13, part e) we did $12 \div \frac{3}{4} = 16$

this quotient is NOT smaller than divisor ($\frac{3}{4}$ in this case) or dividend (12 in this case).

So, if we divide a number by a fraction which is less than 1, then we will get a bigger number. Think in terms of the number of pieces you will get, not size of the pieces in the following picture.



$4 \div \frac{1}{2} \rightarrow$ how many $\frac{1}{2}$ apples make 4 apples

8 half apples make 4 apples, So $4 \div \frac{1}{2} = 8$

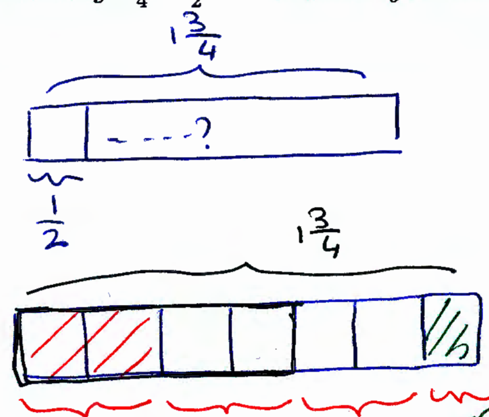
more pieces even though each piece is smaller than 1

[2]Idea taken from Tirosh and Graeber (1990), *Evoking Cognitive Conflict to Explore Preservice Teachers' Thinking About Mathematics*, JRME, Vol.21, No. 2, 98–108.

Problem 44. *People have different approaches to solving $1\frac{3}{4} \div \frac{1}{2}$. How would you solve it?*[3]

$1\frac{3}{4} \div \frac{1}{2} \Rightarrow$ How many $\frac{1}{2}$'s make $1\frac{3}{4}$?

(measurement interpretation)

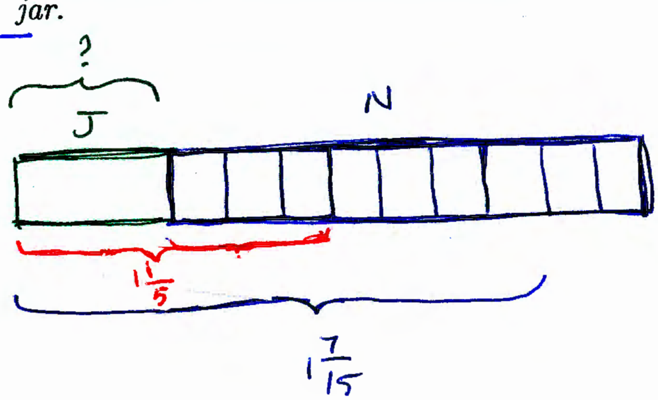


$= 1$ group of $\frac{1}{2}$ $+ 1$ group of $\frac{1}{2}$ $+ 1$ group of $\frac{1}{2}$ $+ \frac{1}{2}$ group of $\frac{1}{2}$ $= 3\frac{1}{2}$

because 1 group of $\frac{1}{2}$ has 2 units and this part has 1 unit, so, half

* Try partitive interpretation by your self !

Problem 45. *(Teacher's Solution)* *Sam bought a big jar of Nutella. When it was $\frac{7}{9}$ full, it weighed $1\frac{7}{15}$ pounds. When it was $\frac{1}{3}$ full, it weighed $1\frac{1}{5}$ pound. Find the weight of the empty jar.*



$1\frac{7}{15} - 1\frac{1}{5} = 4$ units

$1\frac{7}{15} - 1\frac{3}{15} = 4$ units

$\frac{4}{15} = 4$ units

1 unit $= \frac{1}{15}$

3 units $= \frac{3}{15}$

J + 3 units $= 1\frac{1}{5}$

$J + \frac{3}{15} = 1\frac{3}{15}$

$-\frac{3}{15} \qquad -\frac{3}{15}$

$J = 1$

The empty Jar weights 1 pound

[3]Problem taken from Ball (1990), Prospective Elementary and Secondary Teachers' Understanding of Division, JRME, Vol.21, No. 2, 132–144.

Problem 46. *Discuss the pros and cons of using a visual model versus using the "invert and multiply" algorithm for division of fractions.*

* conceptually understanding
* developmen of Structure in mind

→ rote

* memorization
* when you forget the rule, you'll forget everything.
* but knowing them with understanding makes life easy

Problem 47. *On a separate piece of paper, please write your name and a definition for fractions, keeping in mind all the concepts about fractions that were discussed in the past few classes.*

Definition of Fraction

DEFINITION (Def 6.2 from the text): Each *fraction* is a multiple of a *fractional unit*, specifically: $\frac{a}{b} = a \cdot \frac{1}{b}$, where a and b are whole numbers, and $b \neq 0$.

multiplicative inverse Property

DEFINITION (Multiplicative Inverse Property from p. 162 of the text.) For each nonzero fraction x there is a unique fraction called the *inverse*, written $\frac{1}{x}$, which satisfies

$$x \cdot \frac{1}{x} = 1.$$

[eg. $3 \cdot \frac{1}{3} = \frac{3}{3} = 1$]

4 Arithmetic Properties

For all real numbers x, y, and z (in particular, if x,y,z are fractions), the following properties are always true:

- Commutative: $x + y = y + x$ and $x \cdot y = y \cdot x$ [$\frac{1}{3} + \frac{2}{3} = \frac{2}{3} + \frac{1}{3} = \frac{1+2}{3}$]
- Associative: $x + (y + z) = (x + y) + z$ and $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ [$\frac{1}{3} + (\frac{2}{5} + \frac{1}{5})$] = [$(\frac{1}{3} + \frac{2}{5}) + \frac{1}{5}$]
- Distributive: $x \cdot (y \pm z) = x \cdot y \pm x \cdot z$
- Additive and Multiplicative Identities: $x + 0 = x$ and $x \cdot 1 = x$
- Additive and Multiplicative Inverse: $x + (-x) = 0$, and if $x \neq 0$ then $x \cdot \frac{1}{x} = 1$

[$\frac{1}{3} + (-\frac{1}{3}) = \frac{1-1}{3} = \frac{0}{3} = 0$]

5 Properties of equality

For all real numbers x, y, and z (in particular, if x, y, z are fractions), the following properties are always true:

i) $x = x$.

ii) Commutative: If $x = y$ then $y = x$.

iii) Transitive: if $x = y$ and $y = z$ then $x = z$. → [eg $\frac{1}{2} = \frac{2}{4}$ & $\frac{2}{4} = \frac{3}{6}$ ⇒ $\frac{1}{2} = \frac{3}{6}$]

iv) If $x = y$ then $x + z = y + z$, $x - z = y - z$, $x \cdot z = y \cdot z$, and if $z \neq 0$ then $x \div z = y \div z$.

replace x by y

5.1 Fraction Rules

Lemma (Multiplication of fractional units): Prove using picture and algebraic methods: $\dfrac{1}{a} \cdot \dfrac{1}{b} = \dfrac{1}{a \cdot b}$

For all whole numbers a, b, c, d, n PROVE (assuming only the definitions, previous lemma, above properties, and what you prove from here on):

- $\frac{a}{b} = \frac{an}{bn}$ where b and n are not zero.

- $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$ or $\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm cb}{bd}$

Simplify the following complex fractions

Problem 48.

$$\frac{\frac{\frac{5}{2}}{\frac{10}{7}}}{\frac{11}{8}} = \frac{\frac{5}{2}\div\frac{10}{7}}{\frac{11}{8}} = \frac{\frac{\cancel{5}^{1}}{2}\cdot\frac{7}{\cancel{10}_{2}}}{\frac{11}{8}} = \frac{\frac{7}{4}}{\frac{11}{8}} = \frac{7}{4}\div\frac{11}{8} = \frac{7}{\cancel{4}_{1}}\cdot\frac{\cancel{8}^{2}}{11} = \frac{14}{11}$$

$$\frac{\frac{5}{2}}{\frac{\frac{10}{7}}{\frac{11}{8}}} = \frac{\frac{5}{2}}{\frac{10}{7}\div\frac{11}{8}} = \frac{\frac{5}{2}}{\frac{10}{7}\cdot\frac{8}{11}} = \frac{\frac{5}{2}}{\frac{80}{77}} \qquad \frac{5}{2}\div\frac{80}{77} = \frac{\cancel{5}^{1}}{2}\cdot\frac{77}{\cancel{80}_{16}} = \frac{77}{32}$$

$$\frac{\frac{3^{10}\cdot5^{4}\cdot7^{8}}{21^{3}\cdot9^{3}}}{\frac{15^{6}\cdot21^{2}\cdot35^{4}}{9^{8}\cdot5^{3}}} = \frac{\frac{3^{10}\cdot5^{4}\cdot7^{8}}{3^{3}\cdot7^{3}\cdot3^{6}}}{\frac{3^{6}\cdot5^{6}\cdot3^{2}\cdot7^{2}\cdot5^{4}\cdot7^{4}}{3^{16}\cdot5^{3}}} = \frac{\frac{3^{10}\cdot5^{4}\cdot7^{8}}{3^{9}\cdot7^{3}}}{\frac{3^{8}\cdot5^{10}\cdot7^{6}}{3^{16}\cdot5^{3}}} = \frac{3^{10-9}\cdot5^{4}\cdot7^{8-3}}{3^{8-16}\cdot5^{10-3}\cdot7^{6}}$$

$$= \frac{3^{1}\cdot5^{4}\cdot7^{5}}{3^{-8}\cdot5^{7}\cdot7^{6}}$$

$$= 3^{1-(-8)}\cdot5^{4-7}\cdot7^{5-6}$$

$$= 3^{9}\cdot5^{-3}\cdot7^{-1} = \frac{3^{5}}{5^{3}\cdot7}$$

$$\frac{6\frac{7}{12}-\left(3\frac{1}{16}-2\frac{1}{8}\right)}{15\frac{3}{4}+\left(2\frac{1}{2}-1\frac{5}{6}\right)} = \frac{6\frac{7}{12}-\left(3\frac{1}{16}-\frac{2}{16}\right)}{15\frac{3}{4}+\left(2\frac{3}{6}-1\frac{5}{6}\right)} = \frac{6\frac{7}{12}-\left(2\frac{17}{16}-\frac{2}{16}\right)}{15\frac{3}{4}+\left(1\frac{9}{6}-1\frac{5}{6}\right)} = \frac{6\frac{7}{12}-2\frac{15}{16}}{15\frac{3}{4}+\frac{4}{6}}$$

$$= \frac{6\frac{28}{48}-2\frac{45}{48}}{15\frac{9}{12}+\frac{8}{12}} = \frac{5\frac{76}{48}-2\frac{45}{48}}{15\frac{17}{12}} = \frac{3\frac{31}{48}}{16\frac{5}{12}} = 3\frac{31}{48}\div16\frac{5}{12} = \frac{2100}{9456}$$

[convert improper fraction and invert & multiply]

Problem 49. *Simplify using fraction arithmetic. Be sure to give the reason for each equality by citing the rule or property used.*

$$\frac{4}{21} \div \frac{5}{4} - \frac{3}{5} \div \frac{7}{8}$$

$$\frac{4}{21} \cdot \frac{4}{5} - \frac{3}{5} \cdot \frac{8}{7} = \frac{16}{105} - \frac{24}{35}$$ [Invert & multiply]

$$= \frac{16}{105} - \frac{24 \cdot 3}{35 \cdot 3}$$ [equivalent fractions - Rule 1]

$$= \frac{16}{105} - \frac{72}{105}$$

$$= \frac{16-72}{105}$$ [Rule 2]

$$= \frac{-56}{105}$$

Problem 50. *Do the computations, leaving your answer as a fraction in simplest form.*

$$[(\tfrac{x}{y} \cdot \tfrac{z}{y}) + (\tfrac{y}{z} \div \tfrac{x}{y})] \div \tfrac{z}{x}$$

$$\left[\frac{xz}{y^2} + \left(\frac{y}{z} \cdot \frac{y}{x}\right)\right] \div \frac{z}{x}$$ [invert & multiply]

$$\left[\frac{xz}{y^2} + \frac{y^2}{xz}\right] \div \frac{z}{x} = \left[\frac{xz \cdot xz + y^2 \cdot y^2}{y^2 \cdot xz \quad xz \cdot y^2}\right] \div \frac{z}{x}$$ [equivalent fractions Rule-1]

$$= \frac{(xz)^2 + y^2}{xz \cdot y^2} \cdot \frac{x}{z} = \frac{\cancel{x}(x^2z^2 + y^2)}{z \cdot \cancel{x}z \cdot y^2}$$

$$= \frac{x^2z^2 + y^2}{z^2y^2}$$