

Test 3
T101 Spring 2014
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Name: KEY

In all problems you must **show your work** to receive credit (except for short answer problems). No books, notes, calculators, computers, or cellphones are allowed. Please try to fit your answer in the space provided, and do not use any additional paper of your own. If you need more space you can continue on the back of the same page; please indicate clearly that your answer is continued on the back.

Question	Points	Question	Points	Question	Points
1	/12	2	/16	3	/16
4	/12	5	/12	6	/16
7	/16				

1. Answer the following questions:

(a) (4 points) Find $\text{LCM}(2^4 \times 5^4 \times 7, 2^7 \times 3 \times 5^2)$ (leave it as a prime factorization if you like).

$$2^7 \cdot 3 \cdot 5^4 \cdot 7$$

(b) (8 points) Use Euclidean Algorithm to find the $\text{GCF}(6992, 3648)$ and then use your answer to simplify $\frac{6992}{3648}$.

Euclidean Algorithm:

$$\begin{array}{r} 1 \\ 3648 \overline{) 6992} \\ \underline{-3648} \\ 3344 \end{array}$$

$$\begin{array}{r} 1 \\ 3344 \overline{) 3648} \\ \underline{-3344} \\ 304 \end{array}$$

$$\begin{array}{r} 11 \\ 304 \overline{) 3344} \\ \underline{-304} \\ 304 \\ \underline{-304} \\ 0 \end{array}$$

Hence, $\text{GCF}(6992, 3648) = 304$.

Now,

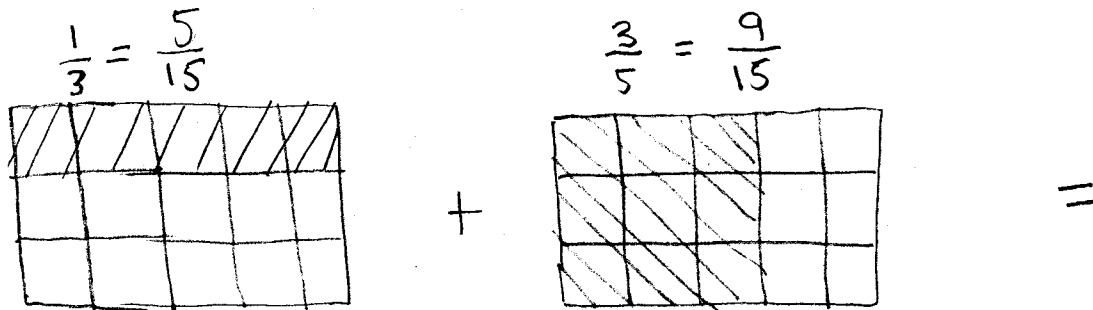
$$\begin{array}{r} 23 \\ 304 \overline{) 6992} \\ \underline{-608} \\ 912 \\ \underline{-912} \\ 0 \end{array} \Rightarrow 6992 = 23 \times 304$$

$$\begin{array}{r} 12 \\ 304 \overline{) 3648} \\ \underline{-304} \\ 608 \\ \underline{-608} \\ 0 \end{array} \Rightarrow 3648 = 12 \times 304$$

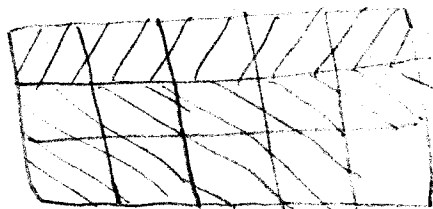
Hence, we can simplify : $\frac{6992}{3648} = \frac{23 \times 304}{12 \times 304} = \frac{23}{12} \cdot \frac{304}{304} = \frac{23}{12} \cdot 1 = \boxed{\frac{23}{12}}$

2. Use an area model to illustrate each arithmetic operation. Show the accompanying SCA as you compute each value.

(a) (8 points) $\frac{1}{3} + \frac{3}{5}$.

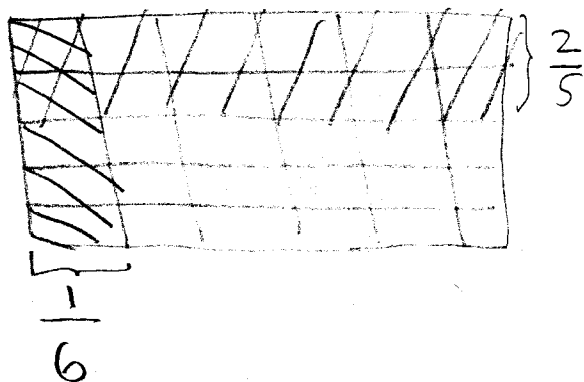


$$\frac{5}{15} + \frac{9}{15} = \frac{14}{15}$$



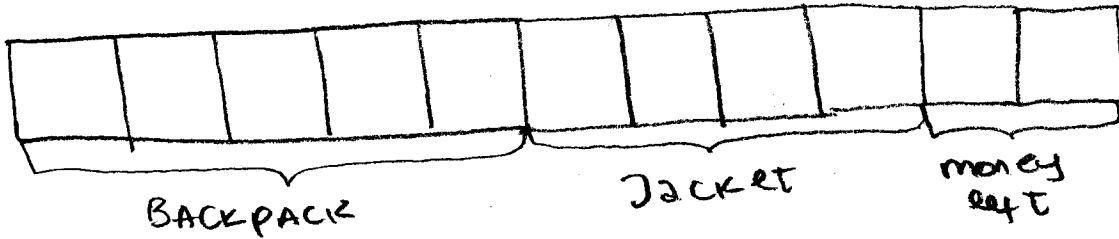
(b) (8 points) $\frac{2}{5} \times \frac{1}{6}$.

$$\frac{2}{5} \times \frac{1}{6} = \frac{2}{30}$$



3. Give Teacher's solutions for the following word problems.

(a) (8 points) Alice spent $\frac{5}{11}$ of her money on a backpack and \$40 on a jacket. She had $\frac{2}{11}$ left. How much did she have to begin with and how much she spent on back pack?

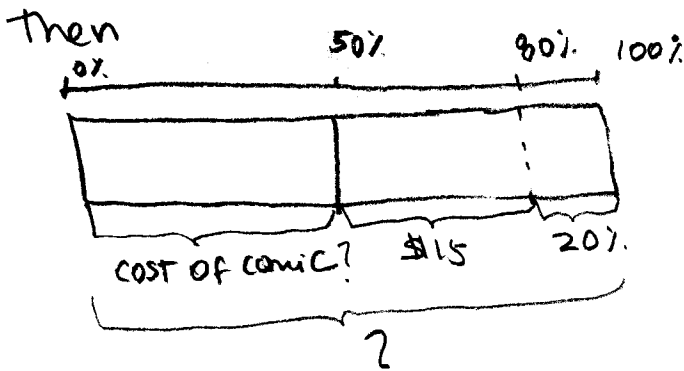


$\div 4 \left\{ \begin{aligned} 4 \text{ units} &= \$40 \\ 1 \text{ unit} &= \$10 \\ 11 \text{ units} &= \$110 \\ 5 \text{ units} &= \$50 \end{aligned} \right.$

Alice had \$110 to begin with and she spent \$50 on backpack.

(b) (8 points) Sofia has a sum of money that she is using to buy two equally priced comic books. If she tries to buy both, she will be short of 20% of the cost. If she only buys one, she will have \$15 left. Find the sum of money that Sofia has and the comic's cost.

Let the whole unit be the total cost for the two comics.



$$\$15 \rightarrow 80\% - 50\%$$

$$\$15 \rightarrow 30\% \downarrow \div 3$$

$$\$5 \rightarrow 10\% \downarrow \times 5$$

$$\$25 \rightarrow 50\%$$

$$\$25 + \$15 = \$40$$

Sofia's sum of money is \$40.

The cost of a comic book is \$25.

4. (12 points) Carlos is a soccer fan. A true fan always buys and tries to fill the entire World Cup album. Suppose that Carlos bought the album for this year's World cup and a box of stickers. After placing stickers in the appropriate players' placeholders he realized that there are 280 empty placeholders. Suppose also that there are a total of 640 placeholders in the entire album.

(a) What is the ratio of empty to filled placeholders in simplest form?

$$280 : (640 - 280) = 280 : 360 = 28 : 36 = \boxed{7:9}$$

For parts b, c and d, simplify your answer whenever possible.

(b) Express the number of empty placeholders as a fraction of the total number of placeholders.

$$\frac{280}{640} = \frac{28}{64} = \boxed{\frac{7}{16}}$$

(c) Express the number of empty placeholders as a fraction of the number of filled placeholders.

$$\frac{280}{360} = \frac{28}{36} = \boxed{\frac{7}{9}}$$

(d) Express the number of filled placeholders as a fraction of the total number of placeholders.

$$\frac{360}{640} = \frac{36}{64} = \boxed{\frac{9}{16}}$$

5. (12 points) Decide if the following statement is True or False, and give a proof or a counterexample accordingly:

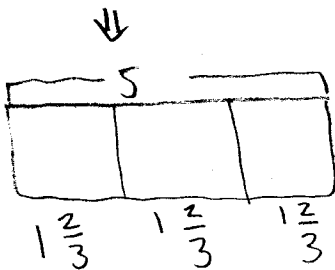
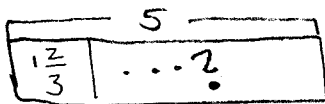
For any integers a and b , if $a < b$ then $a^2 < b^2$.

FALSE. Consider: $a = -2$ and $b = 1$. then
 $a = -2 < 1 = b$, but $a^2 = (-2)^2 = 4 > 1 = b^2$

6. For each of the following, pose the interpretive (characteristic) question for the following division problem, illustrate using a bar diagram, and solve the problem without using the SCA.

(a) (8 points) Measurement division for $5 \div 1\frac{2}{3}$

Q: How many $1\frac{2}{3}$'s make 5?



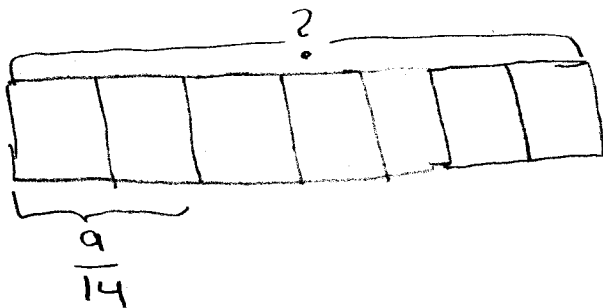
So 3 groups of $1\frac{2}{3}$ make 5, i.e.,

$$5 \div 1\frac{2}{3} = 3$$

$$\left[\begin{array}{l} 1 + \frac{2}{3} + 1 + \frac{2}{3} + 1 + \frac{2}{3} = \\ 3 + \frac{6}{3} = 3 + 2 = 5 \end{array} \right]$$

(b) (8 points) Partitive division for $\frac{9}{14} \div \frac{2}{7}$

Q: $\frac{9}{14}$ is $\frac{2}{7}$ of what size?



$$2 \text{ units} = \frac{9}{14} \quad \downarrow \div 2$$

$$1 \text{ unit} = \frac{9}{28} \quad \downarrow \times 7$$

$$7 \text{ units} = \frac{9}{4}$$

So $\frac{9}{14}$ is $\frac{2}{7}$ of $\frac{9}{4}$, i.e.,

$$\frac{9}{14} \div \frac{2}{7} = \frac{9}{4}$$

7. (16 points) Find integers m and n such that $4273m + 1422n = 1$. Show your work step-by-step and clearly to receive full credit. DO NOT USE GUESS AND CHECK METHOD!

Using the extended Euclidean Algorithm, let us find $\text{GCF}(4273, 1422)$:

$$\begin{array}{r} 3 \\ 1422 \overline{) 4273} \\ \underline{-4266} \\ 0007 \end{array}$$

↓

$$4273 = 1422 \times 3 + 7$$

↓

$$7 = 4273 - 3 \times 1422$$

$$\begin{array}{r} 203 \\ 7 \overline{) 1422} \\ \underline{-14} \\ 022 \\ \underline{-21} \\ 1 \end{array}$$

↓

$$1422 = 203 \times 7 + 1$$

↓

$$1 = 1422 - 7 \times 203$$

$$\begin{array}{r} 7 \\ 1 \overline{) 7} \\ \underline{-7} \\ 0 \\ \boxed{1} \end{array}$$

Hence,
 $\text{GCF}(4273, 1422) = 1$.

$$1 = 1422 - (4273 - 3 \times 1422) \times 203$$

$$1 = 1422 - 203 \times 4273 + 609 \times 1422$$

$$\boxed{1 = 610 \times 1422 - 203 \times 4273}$$

Hence,

$$m = -203$$

$$n = 610$$