Name ______________________ December 17, 2003

Math T101 Final Exam

Do all 27 problems (200 points total). Place your answers in the spaces provided. You must show your work to receive credit. No calculators allowed.

(1) (5 points) Express the fraction in simplest form.

$$\frac{\frac{3}{5}-\frac{1}{4}}{1+\frac{2}{5}} =$$

(2) (5 points) Calculate $0.02 \div 0.0016 =$

(3) (5 points) Simplify as much as possible. Write out ***every*** step neatly – this will reduce errors.

$$\frac{6^{36} \cdot 3^{-16}}{12^{18} \cdot 1024^{0}}$$

(4) (6 points) Circle the numbers which divide $n = 129,030$

2 3 4 5 8 9 10 11

(5) (5 points) Give a correctly labeled picture proof that an odd plus an odd number is an even number.

(6) (6 points) Use the part-whole interpretation of subtraction and a number line to illustrate $6 - (-3) = 9$.

(7) (4 points each) **Show** how to compute the following **mentally** (eg. for $98 + 24$ write $= 100 + 22 = 122$)

(a) Use compensation to find $892 - 189 =$

(b) $47 \times 99 \;=$

(c) $4160 \div 5 \;=$

(d) $0.708 \times .36 + (-2.36 \times 0.708) \;=$

(e) Estimate using compatible numbers: $1270 \div 39 \;\approx$

(8) ($8 = 4 + 2 + 2$ points) (a) State the two main stages in teaching the Addition Algorithm. Give a numerical example for each stage.

(b) (Fill in the blank) The two stages of learning the Addition Algorithm are determined by

the ____________________ process (i.e., the stages are determined by the mathematics!).

(c) Give one goal for teaching Mental Math.

(9) (6 points) Illustrate the subtraction algorithm using the chip model for the following problem.

$$\begin{array}{r} 402 \\ -\ 234 \\ \hline \end{array}$$

(10) (5 points) Here are five numbers. Circle the numbers which can be represented by finite decimals.

$$\frac{18}{15} \qquad \sqrt{18} \qquad \frac{62}{125} \qquad \sqrt{121} \qquad \frac{5}{12}$$

(11) (6 points) Illustrate (and find!) the answer to $5\frac{1}{3} - 2\frac{3}{4}$ using a measurement model.

(12) (6 points) Draw an area model for $\frac{1}{3} + \frac{2}{5}$.

(13) (3 points each) Identify whether the following problems are using measurement division (MD) or partitive division (PD). **Do not solve**.

1. Jane poured 4 cups of milk equally into 7 glasses. How much was in each glass? __________
2. If the $36 cost of dinner is split equally among 8 people, how much does each pay? __________
3. If each serving of yogurt requires $\frac{1}{2}$ cup of milk, how many servings will 7 cups of milk make? __________

(14) (5 points) Write down an irrational number between 0.91 and 0.92. Make your notation clear.

(15) (6 points) Consider the repeating decimal $x = 0.1238238238\ldots$. Is x rational? yes no (circle one). If so, what fraction does it represent?

(16) (*4 points*) Put the numbers 1,2,3, and 4 in the blanks to indicate the correct teaching order:

(a) fraction ÷ fraction __________

(b) whole number ÷ fraction __________

(c) fraction ÷ whole number __________

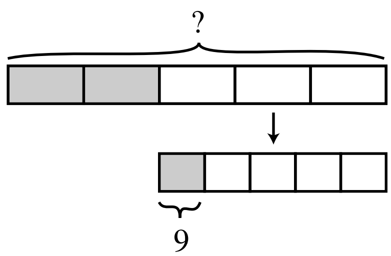
(d) whole number ÷ whole number __________

(17) (10=4+4+2 points) (a) Draw a bar diagram for the problem $36 \div \frac{3}{4}$ using partitive division.

(b) Give a detailed explanation of how to use the diagram to find $36 \div \frac{3}{4}$.

(c) Verify that you get the same answer by 'inverting and multiplying'.

(18) (6 points) Make up a two-step fraction word problem such that the bar diagram in the teacher's solution is:



(19) (5 points each) (a) Find the prime factorization of 3564.

(b) Is 247 prime? Why or why not?

(20) (9 points) There are 3 times as many girls as boys in a school choir.

(a) Draw a bar diagram for this situation.

(b) What is the ratio of the number of girls to the total number of children? __________

(c) What fraction of the children are boys? __________

(d) If there are 27 girls, how many children are there altogether? __________

Give a Teacher's Solution to the following five word problems.

(21) (8 points) John is 15 lbs heavier than Peter. Their total weight is 127 lbs. Find John's weight.

(22) (8 points) Susan bought a television on sale for \$140. This was 70% of its usual price. What was the usual price of the television?

(23) (8 points) A sum of money was shared between Jane and Nancy in the ratio 3 : 11. Nancy received \$248 more than Jane. If Nancy decides to give \$31 to Jane, find the new ratio of Jane's money to Nancy's money.

(24) (8 points) A tank is $\frac{3}{5}$ full of water. When 500 $m\ell$ of water is poured out, the tank becomes $\frac{1}{2}$ full. Find the capacity of the tank.

(25) (8 points) Mr Li made some tarts. He sold $\frac{3}{5}$ of them and gave $\frac{1}{4}$ *of the remainder* to his friends. If he had 150 tarts left, how many tarts did he sell?

(26) (8 points) Illustrate the identity $(a+b)(c+d) = ac + ad + bc + bd$ using a rectangular array. Show how to derive the identity using the distributive property.

(27) (10 points) Prove **ONE** of the following three statements. *Make clear which one you choose — only one will be graded.*

(1) (a) $x = 0.1010010001\cdots$ is an irrational number. Show that $5x$ is also an irrational number.
(b) Prove that if x is an irrational number, then so is $\frac{a}{b}x$.

(2) (a) State the Additive Inverse Property of the integers.
(b) Use the Additive Inverse and other properties of the integers to prove that $-(-a) = a$.

(3) (a) Show why $17^4 \cdot 13^4 = 221^4$. (explain why, do not just use the rule!)
(b) Prove that $a^m \cdot b^m = (ab)^m$ for any positive whole numbers a, b, and m.